Trapped-Particle Asymmetry Modes in Single-Species Plasmas

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Novel trapped-particle asymmetry modes propagate on cylindrical electron columns when axial variations in the wall voltage cause particle trapping. These modes consist of $\mathbf{E} \times \mathbf{B}$ drifts of edge-trapped particles, partially shielded by axial flows of interior untrapped particles. A simple model agrees well with the observed frequencies and eigenfunctions, but the strong mode damping is as yet unexplained. These modes may be important in coupling trap asymmetries to particle motions and low frequency $\mathbf{E} \times \mathbf{B}$ drift modes.

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Single-species plasmas confined in cylindrical traps with static electric and magnetic fields are utilized in research ranging from basic plasma and fluid dynamics to spectroscopic frequency standards to antimatter containment [1]. The plasma confinement times can be hours or days, generally limited by azimuthal asymmetries in the trapping fields, which couple angular momentum into the rotating plasma [2]. This coupling can be enhanced by mode resonances [3], and plasma modes occur on frequency scales ranging from the (high) cyclotron frequency $\Omega_c$, to the plasma frequency $\omega_p$, to the (low) $\mathbf{E} \times \mathbf{B}$ drift frequency $\omega_E$. However, the well-studied “diocotron” drift modes have axial wave number $k_z = 0$, and so do not readily couple the plasma electrostatic energy into particle kinetic energy.

Here, we describe novel “trapped-particle asymmetry” modes that propagate azimuthally at low $\mathbf{E} \times \mathbf{B}$ drift frequencies but that have $k_z \neq 0$ and thus couple to particle kinetics. These modes exist when axial variations in the wall potential cause the equilibrium plasma to have axially trapped particles. Experimentally, we generate the trapped particles by applying a “squeeze” voltage to a central cylindrical electrode, but smaller trapping potentials are caused by wall irregularities and are probably endemic to these traps.

These asymmetry modes can have an azimuthal mode number $m = 1, 2, \ldots$, but we focus on $m = 1$ here. Dynamically, the modes are an $\mathbf{E} \times \mathbf{B}$ drift motion of the edge-trapped particles with axial flows of interior untrapped particles giving partial Debye shielding. A simple theory model with these characteristics shows close correspondence with the measured frequencies and radial eigenfunctions of the modes. Experimentally, the modes are observed to be exponentially damped, but this damping is not yet understood. These modes are particularly simple cylindrical analogs to the trapped drift modes which can contribute to anomalous plasma transport in tokamak-like and toroidal multipole configurations of neutral plasmas [4,5].

The cylindrical confinement geometry and a schematic of the mode are shown in Fig. 1. A nominally cylindrical column of electrons emitted from a hot tungsten source is confined radially by a uniform magnetic field $B = 4$ kG and confined axially by negative voltages $-V_c = -100$ V on end cylinders. Typical electron columns have a central density $n_0 = 1.5 \times 10^{17}$ cm$^{-3}$ over a length $L_p \approx 50$ cm, with a column radius of $R_p = 1.2$ cm inside a wall radius $R_w = 3.5$ cm. The $z$-averaged electron density $n(r, \theta, t)$ can be (destructively) measured at any time by dumping the electrons axially onto a phosphor screen imaged by a CCD camera [6].

The individual electrons have a thermal energy $T = 1$ eV, giving an axial bounce frequency $f_b = \bar{v}/2L_p = 0.5$ MHz and a Debye shielding length $\lambda_D = (T/4\pi e^2 n)^{1/2} = 0.2$ cm. The unneutralized electron charge results in a central potential $-\phi_0 = -30$ V, and the radial electric field causes the column to $\mathbf{E} \times \mathbf{B}$ drift rotate at a rate $f_E(r) = cE(r)/2\pi r B \approx 50$ kHz. Figure 2 shows typical radial profiles of density and rotation for the quiescent, $\theta$-symmetric, quasi-steady-state columns on which we study the waves.

When a negative squeeze voltage $-V_{sq}$ is applied to a central cylinder, electrons are excluded from the column periphery under the squeeze ring, and those electrons located at radii at $r > r_s$ are trapped axially in one end or the other. For small $V_{sq}$, a fraction $e_{tr} = V_{sq}/\phi_0$ of all the electrons are trapped axially.

Linear $\mathbf{E} \times \mathbf{B}$ drift modes with azimuthal mode numbers $m = 1, 2, \ldots$ propagate on this trapped-particle equilibrium. The ubiquitous $m = 1$ center-of-mass diocotron

FIG. 1. Schematic of the cylindrical trap and squeezed electron column with asymmetry mode, showing the positive and negative trapped and untrapped density perturbations.
mode is essentially uniform in $z$ (i.e., $k_z = 0$) and is nominally unaffected by small $V_{sq}$. In contrast, the $m = 1$ trapped-particle asymmetry mode described here has odd parity in $z$: the perturbations are essentially uniform within each end but of opposite sign on either side of the squeeze, as shown schematically in Fig. 1. Moreover, the perturbations in the trapped particles at $r > r_s$ are partially “shielded” by perturbations of opposite sign in the untrapped particles at $r < r_s$, and these untrapped particles slosh from end to end in response to the $E \times B$ drift evolution of the trapped particles.

Experimentally, these odd-$z$-parity $m = 1$ asymmetry modes are excited with a short burst of 1–10 sinusoidal oscillations which are phased + and − on $\theta$-opposite and $z$-opposite wall sectors, as shown in Fig. 1. The asymmetry mode could actually be excited from any single sector, but the configuration above minimizes the concurrent excitation of other modes. Any single wall sector can be used as a receiver, or sectors can be used in combination to verify the $\theta$ and $z$ symmetries of the modes.

Figure 3 shows the measured frequencies $f_a$ and damping rates $\gamma_a$ for the trapped-particle asymmetry mode as the applied squeeze voltage $V_{sq}$ is varied. The frequency $f_d$ of the diocotron mode is also shown for reference. The asymmetry mode frequency is at or near the edge rotation frequency $f_E(R_p)$ for $V_{sq} \ll \phi_0$. As $V_{sq}$ is increased, the trapping separatrix $r_s$ moves inward, and $f_a$ decreases. For $V_{sq} \gtrsim \phi_0$, the column is cut in half, and the asymmetry mode becomes degenerate with the diocotron mode. With all particles trapped, the asymmetry mode is equivalent to two separate diocotron modes, 180° out of phase. The diocotron frequency $f_d$ increases slightly with $V_{sq}$, because the effective line density of the column increases as particles are excluded from the squeeze ring. Both $f_a$ and $f_d$ scale as $f \propto B^{-1}$, as expected for $E \times B$ drift modes.

The asymmetry mode is observed to damp exponentially with time; that is, the received wall signals are basically sinusoidal, decreasing in amplitude as $\exp(-\gamma_a t)$. The damping rate $\gamma_a$ depends on $V_{sq}$ decreasing roughly as $(f_a - f_d)$ in the regime $V_{sq} \gtrsim \phi_0/10$ where $\gamma_a$ can be readily measured. Figure 3 shows that $\gamma_a/f_a \approx 1/20$, so the mode is strongly damped except at $V_{sq} \gtrsim \phi_0$. However, we observe essentially no magnetic field dependence to the damping in the range of 0.5 kG $\leq B \leq 10$ kG, i.e., $\gamma_a \propto B^0$. Thus, $\gamma_a/f_a \propto B^1$ and the mode is relatively less strongly damped at lower magnetic fields.

As yet, the cause of this damping is not understood, but experimental signatures suggest diffusive mixing between the trapped and untrapped populations. Also, for $V_{sq} < \phi_0/10$, there are suggestions that spatial Landau damping (scaling as $B^{-1}$) may occur, but this damping depends critically on the density profile near $R_p$.

The density eigenfunctions $\delta n_a(r)$ of the asymmetry mode can be obtained from a time sequence of measurements of the density $n_h(r, \theta, t) z$ averaged over only half the plasma. Here, time $t = 0$ is defined by the wave excitation, so the measured $n_h$ is synchronous with the wave. The density perturbations associated with each mode are essentially independent of $z$ over each half of the plasma, but the asymmetry mode perturbation changes sign at the squeeze barrier. To dump only half the column onto the phosphor screen, $V_{sq}$ is increased to 100 V immediately before lowering $E_c$.

The $m = 1$ perturbations in $n_h(r, \theta, t)$ are then fit to a sum of two modes (asymmetry and diocotron), as

$$\int d\theta n_h(r, \theta, t)e^{-i\omega t} = \sum_{j=a,d} \delta n_j(r)e^{i2\pi f_j t - \gamma_j t}.$$

The residual to this fit is small, so the eigenfunctions are obtained with little ambiguity, and the $f_j$ and $\gamma_j$ match.
the wall sector time sequence data. The real part of the asymmetry mode eigenfunction $\delta n_a$ (scaled arbitrarily) obtained for $V_{sq} = 5$ V is shown below $n(r)$ in Fig. 2; with proper choice of $\theta$ origin, the imaginary part of $\delta n_a$ is essentially zero.

The mode shows a negative perturbation at $r > r_s$ and a positive perturbation for $r < r_s$, and the signs of these perturbations reverse for $z \to -z$ and for $\theta \to \theta + \pi$. The observed position of the zero crossing at $r_s$ varies with $V_{sq}$, consistent with the “waistline” radius obtained from $(r, z)$ Poisson solutions for the equilibrium with the measured charge and applied wall potentials.

Fluid theory and kinetic theory models further support the interpretation of the asymmetry mode as trapped $E \times B$ drifting edge particles which are Debye shielded by sloshing interior particles. In these models, the applied squeeze voltage is presumed to create a (zero-length) barrier sloshing interior particles. In these models, the applied squeeze voltage is presumed to create a (zero-length) barrier sloshing interior particles. In these models, the applied squeeze voltage is presumed to create a (zero-length) barrier sloshing interior particles. In these models, the applied squeeze voltage is presumed to create a (zero-length) barrier sloshing interior particles.

Here, $\delta \phi_b \equiv (L_1 \delta \phi_1 + L_2 \delta \phi_2)/(L_1 + L_2)$ is the potential averaged over both sides, and the boundary condition is $\delta \phi_b(R_w) = 0$.

Fortunately, the even and odd axial symmetries of the diocotron and asymmetry modes decouple the equations. For $\delta \phi_1(r) = \delta \phi_2(r) = \delta \phi_b(r)$, both equations reduce to the usual equation for a diocotron mode. For $L_1 \delta \phi_1(r) = -L_2 \delta \phi_2(r)$, giving $\delta \phi_b(r) = 0$, we obtain an eigenvalue equation for the asymmetry mode potentials.

For a uniform density profile of density $n_0$ and radius $R_p$, the eigenfrequency $f = f_a$ is

$$f_a = \frac{f_E(0)}{m} = \frac{2e}{Br} \frac{m \delta \phi_j}{m \delta \phi_b} + \frac{4\pi e^2 n}{T} (\delta \phi_j - \delta \phi_b), \quad r > r_s.$$  

(3)

Here, $I_{m \pm 1} = I_{m \pm 1}(r_s/\lambda_D)$ are modified Bessel functions of the first kind. In the limit of $\lambda_D \to 0$, the interior region acts like a conductor, and the exponentially large $I_{m \pm 1}$ may be replaced by 1 in Eq. (4).

The boundary $r_s$ is determined by radial integration of Poisson’s equation: integration of density $n_0$ out to radius $r_s$ gives $\phi(R_w) = -V_{sq}$. Figure 4 shows $r_s$, $f_a$, and $f_d$ versus $V_{sq}$ for this uniform density model with $R_p = 0.5R_w$ and $m = 1$. There is an obvious general correspondence between this coarse estimate and the experiment (Fig. 3).

A kinetic analysis allowing for trapped and untrapped particles coexisting at each radius gives a more realistic approximation to the experiments. The squeeze causes a potential barrier of strength $\Delta \phi(r)$, giving a trapped-particle density $n_t(r) = n(r) \text{erf}(\Delta \phi / T)^{1/2}$ and an untrapped density $n_u = n - n_t$. The equilibrium Poisson solutions show that $\Delta \phi$ is essentially zero for small radii and that it rises sharply near $r_s$ to a value much larger than $T$. Thus, the kinetic treatment essentially smooths the discrete transition model over a radial scale of a few $\lambda_D$.

The frequency $f_a$ versus $V_{sq}$ predicted by this kinetic theory using the measured $n(r)$ and the calculated $\Delta \phi(r)$ is given by the dashed curve in Fig. 3, showing close agreement with the measured frequencies. The 10% discrepancy

Because the axial bounce frequency is large compared to the $E \times B$ rotation frequency and the mode frequency, the bounce-average density perturbation in any one of the three regions, $\delta n$, is related to the bounce-average potential perturbation in that region, $\delta \phi$, through

$$\delta n(r) = \frac{c}{2\pi Br} \frac{\partial \delta \phi}{\partial r} \frac{m \delta \phi(r)}{m \delta \phi(r) - f}.$$  

(1)

Within any of the regions, rapid axial streaming yields the adiabatic response

$$\delta n(r, z) = \delta n(r) + \frac{en_0}{T} [\delta \phi(r, z) - \delta \phi(r)].$$  

(2)

This type of response gives rise to Debye shielding, making $\delta \phi(r, z)$ and $\delta n(r, z)$ nearly $z$ independent except near the plasma ends and the squeeze region. Thus, we approximate $\delta n(r, z) = \delta n_j(r)$ and $\delta \phi(r, z) = \delta \phi_j(r)$, with $j = 1, 2$ representing the left and right sides relative to the barrier. Poisson’s equation for the two sides is then
may reflect the nonzero length of the barrier and consequent increase in trapped-particle densities.

Similarly, the eigenfunctions $\delta n_0(r)$ obtained from the kinetic analysis show general correspondence with experiments, although the measured eigenfunctions in the untrapped region of $r < r_s$ are smaller than predicted by a factor of approximately $r_s/R_p$. This discrepancy may reflect systematic errors in the dump imaging, or it may be related to particle diffusion and decay of the mode.

Although the asymmetry modes can propagate in a trap with perfect $\theta$ symmetry, our larger interest is in traps with (inevitable) $\theta$ asymmetries. These asymmetries commonly arise from construction imperfections in the trap walls (especially the sectored cylinders) or in the magnet. The asymmetries can have arbitrary $\theta$ and $z$ dependence, but it is common to refer to their dominant $m$ and $k_z$ components. One of the more prevalent asymmetries is a $m = 1$, $k_z = \pi/L_p$ "tilt" of the magnetic field with respect to the trap axis, characterized by an angle $\theta_B \equiv B_\perp/B_z$.

The qualitative behavior of the asymmetry mode remains unchanged even with moderately large magnetic tilt. Figure 5 shows $f_a$ decreasing by 30% and $\gamma_a$ decreasing by 60% at fixed $V_{sq} = -10$ V as the magnetic tilt is increased to $\theta_B = \pm 3 \times 10^{-3}$ rad. These quantitative changes are mainly due to a decrease in the plasma potential $\phi_0$ (caused by tilt-induced particle transport), causing the ratio $V_{sq}/\phi_0$ to increase and the effective trapping barrier to be enhanced.

Trapped particles may be an inherent part of Penning-Malmberg traps for a variety of reasons. Some intentional manipulations, such as "squeeze damping" of the diocotron mode [7,8] obviously create equilibria as described here. The wide variety of "nested traps" being built for overlapping confinement of positrons and antiprotons [9,10] have populations of trapped and untrapped particles. Most subtly, $z$ variations in the effective wall voltage can arise from small unintentional construction anomalies. For example, a variation in wall radius among cylinders gives an effective potential $V_{sq} = \phi_0 \Delta R_w/R_w$: this gives a trapped fraction $\epsilon_{tr} = \Delta R_w/R_w$, so $\epsilon_{tr} \approx 10^{-3}$ is probably common to all devices.

The existence of trapped-particle asymmetry modes substantially alters the theory perspective on particle transport due to $\theta$ asymmetries in the trap construction. Resonant particle transport theories [3] utilize integration along unperturbed orbits, which is invalid with the bulk trapped-particle populations described here, or even with microscopic trapping [11]. Similarly, 2D bounce-averaged invariants suggest that particles are confined to equipotential surfaces, and a $k_z$ variation is required for radial particle transport; however, all previously known modes with $k_z \neq 0$ have frequencies near $\omega_p$ as opposed to $f_a$.

Experimentally, there are strong suggestions that the asymmetry mode contributes to asymmetry-induced transport by coupling between the $k_z \neq 0$ asymmetry and the $k_z = 0$ diocotron mode. For the case of a magnetic tilt $\theta_B$, the diocotron mode has axially sloshing particles [12] at frequency $f_d$ but remains essentially undamped. However, coupling to the axially sloshing particles in the strongly damped asymmetry mode may result in diocotron mode damping and bulk radial transport of particles. The characteristics of this coupling are presently being investigated.

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